CP VIOLATION: A THEORETICAL REVIEW *

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Abstract

This review of CP violation focuses on the status of the subject and its likely future development through experiments in the Kaon system and with B-decays. Although present observations of CP violation are perfectly consistent with the CKM model, we discuss the theoretical and experimental difficulties which must be faced to establish this conclusively. In so doing, theoretical predictions and experimental prospects for detecting $\Delta S = 1$ CP violation through measurements of ϵ'/ϵ and of rare K decays are reviewed. The crucial role that B CP-violating experiments will play in elucidating this issue is emphasized . The importance of looking for evidence for non-CKM CP-violating phases, through a search for a non-vanishing transverse muon polarization in $K_{\mu 3}$ decays, is also stressed.

1 Introduction

The discovery more than 30 years ago of the decay $K_{\rm L} \to 2\pi$ by Christianson, Cronin, Fitch and Turlay [1)] provided the first indication that CP, like parity, was also not a good symmetry of nature. It is rather surprising that, for such a mature subject, we have still so little experimental information available. Indeed, the only firm evidence for CP violation to date remains that deduced from meausurements in the neutral Kaon system. Here there are five parameters meausured: the values of the the two complex amplitude ratios for the decays of $K_{\rm L}$ and $K_{\rm S}$ to $\pi^+\pi^-$ ($n_{+-} = \epsilon + \epsilon'$) and to $\pi^o\pi^o$ ($\eta_{oo} = \epsilon - 2\epsilon'$), plus the semileptonic asymmetry in $K_{\rm L}$ decays ($A_{\rm K_L}$) [1]. However, in fact, these five numbers at present give only one **independent** piece of dynamical information. To a very good approximation [3)], η_{+-} and η_{oo} are equal in magnitude

$$|\eta_{+-}| \simeq |\eta_{oo}| \simeq 2 \times 10^{-3} \ ,$$

and in phase

$$\phi_{+-} \simeq \phi_{oo} \simeq 44^\circ \ ,$$

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[1]CP Lear [2)] has presented recently a preliminary meausurement of the $K_{\rm S}$ semileptonic asymmetry, $A_{\rm K_S}$, which agrees with $A_{\rm K_L}$ within 10%.

with the ratio

$$\epsilon'/\epsilon \leq 10^{-3} \ .$$

Because the two pion intermediate states dominate the neutral Kaon width, assuming CPT conservation [4] the phases ϕ_{+-} and ϕ_{oo} are approximately equal to the superweak phase

$$\phi_{\rm SW} = \tan^{-1} \frac{2\Delta m}{\Gamma_{\rm S} - \Gamma_{\rm L}} = (43.64 \pm 0.14)^\circ \ .$$

CPT conservation also fixes the value of the semileptonic asymmetry in terms of Re ϵ [5]. Since ϵ' is so small, effectively one has [4]

$$A_{\rm K_L} \simeq 2Re\ \eta_{+-}.$$

Thus the dynamical information we have today is, essentially, that obtained in the original discovery experiment [1], augmented by the statement that there is little or no $\Delta S = 1$ CP violation!

The above statement is a bit of an exaggeration, since in the last 30 years we have learned very much more about CP violation **outside** the neutral Kaon complex. In particular, very strong bounds have been established for the electric dipole moments of the neutron and the electron [3]

$$d_e, d_n \leq 10^{-25} \ \mathrm{ecm} \ .$$

Furthermore, we have uncovered the fundamental role that CP violation plays in the Universe, to help establish the observed matter-antimatter asymmetry [6]. In addition, we have a variety of bounds on a host of other CP violating parameters, like the amplitude ratios η_{+-o} and η_{ooo} for $K \to 3\pi$ decays , or the transverse muon polarization $< P_\perp^\mu >$ in $K_{\mu3}$ decays. These bounds, however, are too insensitive to provide much dynamical information.

In the modern gauge theory paradigm CP violation can have one of two possible origins. Either,

i) the full Lagrangian of the theory is CP invariant, but this symmetry is not preserved by the vacuum state: CP $|0\rangle \neq |0\rangle$. In this case CP is a spontaneously broken symmetry [7].

Or

ii) there are terms in the Lagrangian of the theory which are not invariant under CP transformations. CP is explicitly broken by these terms and is no longer a symmetry of the theory.

The first possibility, unfortunately, runs into a potential cosmological problem [8]. As the universe cools below a temperature T^* where spontaneous CP violation occurs, one expects that domains of different CP should form. These domains are separated by walls having a typical surface energy density $\sigma \sim {T^*}^3$. The energy density associated with these walls dissipates slowly as the universe cools further and eventually contributes an energy density to the universe at temperature T of order $\rho_{\rm wall} \sim {T^*}^3 T$. Such an energy density today would typically exceed the universe closure density by many orders of magnitude:

$$\rho_{\rm wall} \sim 10^{-7} \left(\frac{T^*}{\rm TeV} \right)^3 \mathrm{GeV}^{-4} \gg \rho_{\rm closure} \sim 10^{-46} \ \mathrm{GeV}^{-4} \ .$$

One can avoid this difficulty by imaging that the scale where CP is spontaneously violated is very high, so that T^* is above the temperature where inflation occurs. In this case the problem disappears, since the domains get inflated anyway. Nevertheless, there are still problems since it proves difficult to connect this high energy spontaneous breaking of CP with observed phenomena at low energies. What emerges, in general, are models which are quite complex [9], with CP violation being associated with new interactions much as in the original superweak model of Wolfenstein [10].

If, on the other hand, CP is explicitly broken the phenomenology of neutral Kaon CP violation is a quite natural result of the standard model of the electroweak interactions. There is, however, a requirement emerging from the demand of renormalizability which bears mentioning.

Namely, if CP is explicitly broken then renormalizability requires that all the parameters in the Lagrangian which can be complex must be so. A corollary of this observation is that the number of possible CP violating phases in a theory increases with the complexity of the theory, since there are then more terms which can have imaginary coefficients.

In this respect, the three generation ($N_g = 3$) standard model with only one Higgs doublet is the simplest possible model, since it has only one phase. With just one Higgs doublet, the Hermiticity of the scalar potential allows no complex parameters to appear. If CP is not a symmetry, complex Yukawa couplings are, however, allowed. After the breakdown of the $SU(2) \times U(1)$ symmetry, these couplings produce complex mass matrices. Going to a physical basis with real diagonal masses introduces a complex mixing matrix in the charged currents of the theory. For the quark sector, this is the famous Cabibbo-Kobayashi Maskawa (CKM) matrix [11).][2] This $N_g \times N_g$ unitary matrix contains $N_g(N_g-1)/2$ real angles and $N_g(N_g+1)/2$ phases. However, $2N_g - 1$ of these phases can be rotated away by redefinitions of the quark fields leaving only $(N_g-1)(N_g-2)/2$ phases. Thus for $N_g = 3$ the standard model has only one physical complex phase to describe all CP violating phenomena.[3]

If CP is broken explicitly, it follows by the renormalizability corollary that any extensions of the SM will involve further CP violating phases. For instance, if one has two Higgs doublets, Φ_1 and Φ_2, then the Hermiticity of the scalar potential no longer forbids the appearance of complex terms like

$$V = \ldots \mu_{12}\Phi_1^\dagger\Phi_2 + \mu_{12}^*\Phi_2^\dagger\Phi_1 \ .$$

Indeed, if one did not include such terms the presence of complex Yukawa couplings would induce such terms at one loop.

2 CP Violation in the Standard Model: Expectations and Challenges

The gauge sector of the $SU(3) \times SU(2) \times U(1)$ Standard Model contains no explicit phases since the gauge fields are in the adjoint representation, which is real, leading to real gauge couplings. Nevertheless, the nontrivial nature of the gauge theory vacuum [12)] introduces a phase structure (θ vacua [13)]) which allows for the presence of effective CP-violating interactions, involving the non Abelian gauge field strengths and their duals:

$$\mathcal{L}_{\mathrm{CP\ viol.}} = \theta_{\mathrm{strong}}\frac{\alpha_{\mathrm{s}}}{8\pi}F_a^{\mu\nu}\tilde{F}_{a\mu\nu} \ + \ \theta_{\mathrm{weak}}\frac{\alpha_2}{8\pi}W_a^{\mu\nu}\tilde{W}_{a\mu\nu} \ .$$

The weak vacuum angle θ_{weak} is actually irrelevant since the electroweak theory is chiral and through a chiral rotation this angle can be set to zero [14)]. The phase angle θ_{strong}, on the other hand, is problematic. First of all, what contributes physically is not θ_{strong}, since this angle receives additional contributions from the weak interaction sector as a result of the chiral rotations that render the quark mass matrices diagonal. Thus, in reality, the CP-violating efffective interaction is

$$\mathcal{L}_{\mathrm{CP\ viol.}} = \bar{\theta}\frac{\alpha_{\mathrm{s}}}{8\pi}F_a^{\mu\nu}\tilde{F}_{a\mu\nu},$$

where $\bar{\theta} = \theta_{\mathrm{strong}} + \mathrm{Arg\ det\ M}$ and M is the quark mass matrix. The presence of such an interaction in the Standard Model gives rise to a large contribution to the neutron electric dipole moment. One has, approximately [15)],

$$d_n \simeq \frac{e}{M_n}(\frac{m_q}{M_n})\bar{\theta} \ ,$$

[2]If the neutrinos are massless, there is no corresponding matrix in the lepton sector since it can be reabsorbed by redefining the neutrino fields.

[3]This is not quite true. In the SM there is also another phase related to the QCD vacuum angle which leads to a CP violating interaction involving the gluonic field strength and its dual. We return to this point in the next section.

with m_q a typical light quark mass. Thus, to respect the existing experimental bounds on d_n [3], $\bar{\theta}$ must be extremely small:

$$\bar{\theta} \leq 10^{-9} - 10^{-10}.$$

Why this should be so is unclear and constitutes the strong CP problem [16].

There are three possible attitudes one can take regarding the strong CP problem:

i) One can just ignore this problem altogether. Afterall, $\bar{\theta}$ is yet another uncalculable parameter in the Standard Model, no different say than the unexplained ratio $m_e/m_t \sim 10^{-6}$. So why should one worry about this parameter explicitly?

ii) One can try to calculate $\bar{\theta}$ and thereby "explain" the size of the neutron electric dipole moment. To do so, one must imagine that CP is spontaneously broken, so that indeed $\bar{\theta}$ is a finite calculable quantity. However, as was mentioned earlier, then one runs into the domain wall problem. Models that avoid this problem and which, in principle, produce a tiny calculable $\bar{\theta}$ exist. However, they are quite recondite [9] and the price one pays for solving the strong CP problem this way is to introduce considerable hidden underlying structure beneath the Standard Model.

iii) One can try to dynamically remove $\bar{\theta}$ from the theory. This is my favorite solution, which I suggested long ago with Helen Quinn [17]. Quinn and I proposed solving the strong CP problem by imagining that the Standard Model has an additional global chiral symmetry. The presence of this, so called, $U(1)_{PQ}$ symmetry allows one to rotate away $\bar{\theta}$, much as the chiral nature of the $SU(2) \times U(1)$ electroweak theory allows one to rotate away θ_{weak}. However, this solution also requires that axions exist [18] and these elusive particles have yet to be detected [16]! An alternative possibility along this vein is that the Standard Model has a natural chiral symmetry built in, which removes $\bar{\theta}$ because $m_u = 0$ [19].However, this solution appears unlikely, as $m_u = 0$ is disfavored by current algebra analyses [20].

It is fair to say that there is no clear understanding of what to do about the strong CP problem at the moment. My own view is that the existence of this unresolved problem is something that should not be ignored. There is a message here and it may simply be that we do not understand CP violation at all!

In the Standard Model, with one Higgs doublet and three generations of quarks and leptons, besides the strong CP phase $\bar{\theta}$ there is only one other CP-violating angle in the theory. This is the combination of phases in the Yukawa couplings of the quarks to the Higgs doublet which remains after all redefinition of quark fields, leading to a diagonal mass matrix, are done. This weak CP-violating angle appears as a phase in the CKM mixing matrix, V, which details the coupling of the quarks to the charged W-bosons:

$$\mathcal{L}_{CC} = g_2 \bar{u}_i \gamma_\mu (1-\gamma_5) V_{ij} d_j W^\mu \; + \; h.c. \; .$$

However, one does not really know if the complex phase present in the CKM matrix is responsible for the CP violating phenomena observed in the neutral Kaon system. Indeed, one does not know either whether there are further phases besides the CKM phase. Nevertheless, it is remarkable that, as a result of the hierarchial structure of the CKM matrix and of other dynamical circumstances, one can **qualitatively** explain all we know experimentally about CP violation today on the basis of the CKM picture.

2.1 Testing the CKM Paradigm

In what follows, I make use of the CKM matrix in the parametrization adopted by the PDG [3] and expand the three real angles in the manner suggested by Wolfenstein [21] in powers of the sine of the Cabibbo angle λ. To order λ^3 one has then

$$V = \left| \begin{array}{ccc} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ \lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{array} \right|$$

with A, ρ and η being parameters one needs to fix from experiments—with $\eta \neq 0$ signalling CP violation.[4]

This matrix, with is hierarchical interfamily structure, naturally accounts for the three principal pieces of independent information that we have today on CP violation. As we discussed earlier, these are:

i) The value of the mass mixing parameter $|\epsilon| \sim 10^{-3}$, which characterizes the strength of the $K_{\rm L}$ to $K_{\rm S}$ amplitude ratios.

ii) The small value of the ϵ' parameter which typifies direct ($\Delta S = 1$) CP violation, with the ratio $\epsilon'/\epsilon \leq 10^{-3}$.

iii) The very strong bounds on the electric dipole moments of the neutron and the electron, which give $d_e, d_n \leq 10^{-25}$ ecm.

One can "understand" the above three facts quite simply within the Standard Model and the CKM paradigm. In the model the parameter ϵ is determined by the ratio of the imaginary to the real part of the box graph of Fig. 1a. It is easy to check that this ratio is of order

$$\epsilon \sim \lambda^4 \sin\delta \sim 10^{-3} \sin\delta \ .$$

That is, ϵ is naturally small because of the suppression of interfamily mixing without requiring the CKM phase δ to be small.

The explanation of why ϵ'/ϵ is small is a bit more dynamical. Basically, this ratio is suppressed both because of the $\Delta I = 1/2$ rule and because ϵ' arises through the Penguin diagrams of Fig.1b. These diagrams involve the emission of virtual gluons (or photons [5]), which are Zweig suppressed [22)]. Typically this gives

$$\frac{\epsilon'}{\epsilon} \sim \frac{1}{20} \cdot \left[\frac{\alpha_{\rm s}}{12\pi} \ln \frac{m_t^2}{m_c^2}\right] \sim 10^{-3} \ .$$

Finally, in the CKM model the electric dipole moments are small since the first nonvanishing contributions [23)] occur at three loops, as shown in Fig. 1c, leading to the estimate [24)]

$$d_{\rm q,e} \sim e m_{\rm q,e} \left[\frac{\alpha^2 \alpha_{\rm s}}{\pi^3}\right] \left[\frac{m_t^2 m_b^2}{M_{\rm W}^6}\right] \lambda^6 \sin\delta \sim 10^{-32} \text{ ecm} \ .$$

One can, of course, use the precise value of ϵ measured experimentally to determine an allowed region for the parameters entering in the CKM matrix. Because of theoretical uncertainties in evaluating the hadronic matrix element of the $\Delta S = 2$ operator associated with the box graph of Fig. 1a, this parameter space region is rather large. Further restrictions on the allowed values of CKM parameters come from semileptonic B decays and from $B_d - \bar{B}_d$ mixing. Because the parameter A, related to V_{cb}, is better known, it has become traditional to present the result of these analyses as a plot in the $\rho - \eta$ plane. Fig. 2 shows the results of a recent analysis, done in collaboration with my student, K. Wang [25)]. The input parameters used, as well as the range allowed for certain hadronic amplitudes and other CKM matrix elements is detailed in Table 1

The resulting 1σ allowed contour emerging from the overlap of the three constraints coming from ϵ, $B_{\rm d} - \bar{B}_{\rm d}$ mixing and the ratio of $|V_{ub}|/|V_{cb}|$, shown in Fig. 3, gives a roughly symmetric region around $\rho = 0$ within the ranges

$$0.2 \leq \eta \leq 0.5 \ ; \ \ -0.4 \leq \rho \leq 0.4 \ .$$

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[4] It is often convenient instead of using $\rho - i\eta$ to write this in terms of a magnitude and phase: $\rho - i\eta = \sigma e^{-i\delta}$, with δ being the CP violating CKM phase.

[5] The contribution of the electroweak Penguin diagrams are not suppressed by the $\Delta I = 1/2$ rule, but these diagrams are only of $O(\alpha)$, not $O(\alpha_{\rm s})$.

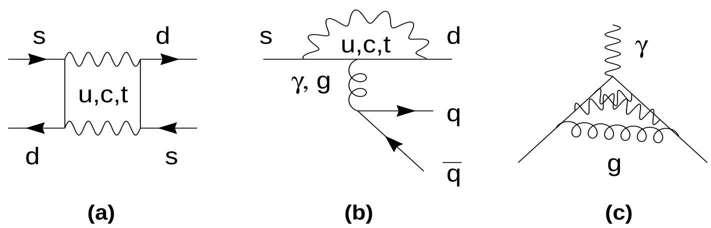

Figure 1: Graphs contributing to ϵ, ϵ' and $d_{\rm q,e}$

Table 1: Parameters used in the $\rho - \eta$ analysis of [25]

$\vert\epsilon\vert$	$=$	$(2.26 \pm 0.02) \times 10^{-3}$	[3]
Δm_d	$=$	$(0.496 \pm 0.032) ps^{-1}$	[26]
m_t	$=$	$(174 \pm 10^{+13}_{-12})$ GeV	[27]
$\vert V_{cb}\vert$	$=$	0.0378 ± 0.0026	[28]
$\vert V_{ub}\vert/\vert V_{cb}\vert$	$=$	0.08 ± 0.02	[28]
$B_{\rm K}$	$=$	0.825 ± 0.035	[29]
$\sqrt{B_d}\ f_{B_d}$	$=$	(180 ± 30) MeV	[30]

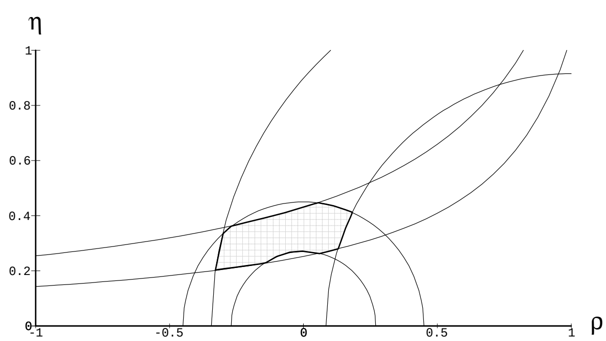

Figure 2: Constraints on the $(\rho,\, \eta)$ plot

As anticipated by our qualitative discussion this region implies that the CKM phase δ is large ($\rho = 0$ corresponds to $\delta = \pi/2$). One should note, however, that this analysis does not establish the CKM paradigm. Using only the B physics constraints one sees that in Fig. 2 there is also an overlap region for $\eta = 0$, which gives $\rho = -0.33 \pm 0.08$ [25]. So one can still imagine that ϵ is due to some other CP violating interaction, as in the superweak model [10], with the CKM phase δ being very small. As Wang and I [25] discussed, one may perhaps eliminate this possibility by improving the bounds on $B_{\rm s} - \bar{B}_{\rm s}$ mixing to $\Delta m_s \geq 10$ ps^{-1}. Since the present LEP bound from ALEPH is $\Delta m_s \geq 6$ ps^{-1} [31], this is not going to be easy. Much more promising, however, is to try to establish the correctedness of the CKM paradigm by looking at further tests of CP violation, both in the Kaon system and by meausuring CP-violating asymmetries in B-decays.

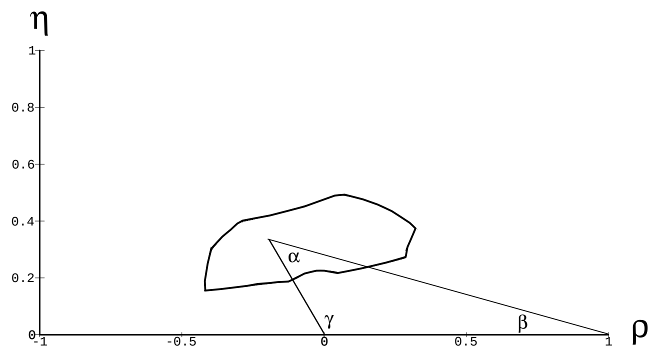

Figure 3: Allowed region in the $\rho - \eta$ plane. Also shown in the plot is a possible unitarity triangle.

In principle, one can obtain quantitative tests of the CKM model purely with Kaon experiments. However, the needed experiments are very challenging, either due to the high precision required or due to the rarity of the processes to be studied. Furthermore, the analysis of these results is also theoretically very difficult, since it requires better estimates of hadronic matrix elements than what we have at present. A good example of both of these challenges is provided by ϵ'/ϵ. The present data on this ratio is inconclusive, with the result obtained at Fermilab [32]:

$$\mathrm{Re}\,\frac{\epsilon'}{\epsilon} = (7.4 \pm 5.2 \pm 2.9) \times 10^{-4} \quad [\mathrm{E731}] \ ,$$

being consistent with zero within the error, while the result of the NA31 experiment at CERN [33] giving a non-zero value to 3σ:

$$\mathrm{Re}\,\frac{\epsilon'}{\epsilon} = (23.0 \pm 3.6 \pm 5.4) \times 10^{-4} \quad [\mathrm{NA31}] \ .$$

Theoretically, the predictions for ϵ'/ϵ are dependent both on the value of the CKM matrix elements and, more importantly, on an estimate of certain hadronic matrix elements.

2.1.1 Prospects for meausuring $\frac{\epsilon'}{\epsilon}$

There has been considerable activity recently to try to narrow down the expectations for ϵ'/ϵ in the Standard Model. To describe the theoretical status here, I find it useful to make use of an approximate formula for this ratio derived by Buras and Lautenbacher [34]. These authors express the real part of this ratio as the sum of two terms [6]

$$\mathrm{Re}\,\frac{\epsilon'}{\epsilon} \simeq 3.6 \times 10^{-3} A^2 \eta \left[B_6 - 0.175 \left(\frac{m_t^2}{M_{\mathrm{W}}^2} \right)^{0.93} B_8 \right] \ .$$

Here B_6 and B_8 are quantities related to the matrix elements of the dominant gluonic and electroweak Penguin operators, respectively. The electroweak Penguin contribution is suppressed relative to the gluonic Penguin contribution by a factor of $\alpha/\alpha_{\mathrm{s}}$. However, as remarked earlier, it does not suffer from the $\Delta I = 3/2$ suppression and so one gains back a factor of about 20. Furthermore, as Flynn and Randall [36] first noted, the contribution of these terms can become significant for large top mass because it grows approximately as m_t^2.

The result of the CKM analysis presented earlier brackets $A^2\eta$ in the range

$$0.12 \leq A^2 \eta \leq 0.31 \ .$$

[6] Because the difference between the $I = 0$ and $I = 2$ $\pi\pi$ phase shifts is also near 45° [35], to a good approximation $\epsilon'/\epsilon \simeq \mathrm{Re}\ \epsilon'/\epsilon$.

For $m_t = 175$ GeV the square bracket in the Buras-Lautenbacher formula reduces to $[B_6 - 0.75 B_8]$. Hence one can write the expectation from theory for ϵ'/ϵ as

$$4.3 \times 10^{-4}[B_6 - 0.75B_8] \leq \mathrm{Re}\ \frac{\epsilon'}{\epsilon} \leq 11.2 \times 10^{-4}[B_6 - 0.75B_8]\ .$$

Because the top mass is so large, the predicted value for ϵ'/ϵ depends rather crucially on **both** B_6 and B_8. These (normalized) matrix elements have been estimated by both lattice [37] and $1/N$ [38] calculations to be equal to each other, with an individual error of $\pm 20\%$:

$$B_6 = B_8 = 1 \pm 0.20\ .$$

Thus, unfortunately, the combination entering in ϵ'/ϵ is poorly known. It appears that the best one can say theoretically is that Re ϵ'/ϵ should be a "few" times 10^{-4}, with a "few" being difficult to pin down more precisely. Theory, at any rate, seems to favor the E731 experimental result over that of NA31.

Fortunately, we may learn something more in this area in the near future. There are 3rd generation experiments in preparation both at Fermilab (KTeV) and CERN (NA48). These experiments should begin taking data in a year or so and are designed to reach statistical and systematic accuracy for ϵ'/ϵ at the level of 10^{-4}. The Frascati Φ factory DAPHNE, which should begin operations in 1997, in principle, can also provide interesting information for ϵ'/ϵ. At DAPHNE one will need an integrated luminosity of $\int \mathcal{L}\, dt = 10$ fb^{-1} to arrive at a statistical sensitivity for ϵ'/ϵ at the level of 10^{-4}. However, if this statistical sensitivity is reached, the systematic uncertainties will be quite different than those at KTeV and NA48, providing a very useful cross-check. It is important to remark that, irrespective of detailed theoretical predictions, the observation of a non-zero value for ϵ'/ϵ at a significant level would provide direct evidence for $\Delta S = 1$ CP violation and would rule out a superweak explanation for the observed CP violation in the neutral K sector.

2.2 Rare Kaon Decays

There are alternatives to the ϵ'/ϵ measurement which could reveal $\Delta S = 1$ (direct) CP violation. However, these alternatives involve daunting experiments [39], which are probably out of reach in the near term. Whether these experiments can (or will?) eventually be carried out is an open question. Nevertheless, it seems worthwhile here to try to outline some of the theoretical expectations for these meausurments.

2.2.1 $K_{\rm S}$ decays

CP Lear already, and DAPHNE soon, will enable a more thorough study of $K_{\rm S}$ decays by more efficient tagging. The decay $K_{\rm S} \to 3\pi^o$ is CP-violating, while the $K_{\rm S} \to \pi^+\pi^-\pi^o$ mode has both CP-conserving and CP-violating pieces. However, even in this case the CP-conserving piece is small and vanishes in the center of the Dalitz plot. Hence one can extract information about CP violation from $K_{\rm S} \to 3\pi$ decays. The analogue $K_{\rm S}/K_{\rm L}$ amplitude ratios to η_{+-} and η_{oo} for $K \to 3\pi$ have both $\Delta S = 1$ and $\Delta S = 2$ pieces:

$$\eta_{ooo} = \epsilon + \epsilon'_{ooo}\ ;\ \ \eta_{+-o} = \epsilon + \epsilon'_{+-o}\ .$$

However, in contrast to what obtains in the $K \to 2\pi$ case, here the $\Delta S = 1$ pieces can be larger. Cheng [40] gives estimates for ϵ'_{+-o}/ϵ and ϵ'_{ooo}/ϵ of $O(10^{-2})$, while others are more pessimistic [41]. Even so, there does not appear to be any realistic prospects in the near future to probe for $\Delta S = 1$ CP-violating amplitudes in $K_{\rm S} \to 3\pi$. For instance, at DAPHNE even with an integrated luminosity of 10 fb^{-1} one can only reach an accuracy for η_{+-o} and η_{ooo} of order 3×10^{-3}, which is at the level of ϵ not ϵ'.

Table 2: Predictions for Asymmetries in $K^{\pm}$ Decays

Asymmetry	Prediction	Φ Factory Reach
$\mathcal{A}(\pi^+\pi^+\pi^-;\pi^-\pi^-\pi^+)$	5×10^{-8} [44]	3×10^{-5}
$\mathcal{A}(\pi^+\pi^o\pi^o;\pi^-\pi^o\pi^o)$	2×10^{-7} [44]	5×10^{-5}
$\mathcal{A}_{\rm Dalitz}(\pi^+\pi^+\pi^-;\pi^-\pi^+\pi^+)$	2×10^{-6} [44]	3×10^{-4}
$\mathcal{A}_{\rm Dalitz}(\pi^+\pi^o\pi^o;\pi^-\pi^o\pi^o)$	1×10^{-6} [44]	2×10^{-4}
$\mathcal{A}(\pi^+\pi^o\gamma;\pi^-\pi^o\gamma)$	10^{-5} [45]	2×10^{-3}

2.2.2 *Asymmetries in charged K-decays*

CP violating effects involving charged Kaons can only be due to $\Delta S=1$ transitions, since $K^+\leftrightarrow K^-$ $\Delta S=2$ mixing is forbidden by charge conservation. A typical CP-violating effect in charged Kaon decays necessitates a comparison between K^+ and K^- processes. However, a CP-violating asymmetry between these processes can occur only if there are at least two decay amplitudes involved and these amplitudes have **both** a relative weak CP-violating phase and a relative strong rescattering phase between each other. Thus the resulting asymmetry necessarily depends on strong dynamics.

To appreciate this fact, imagine writing the decay amplitude for K^+ decay to a final state f^+ as

$$A(K^+\to f^+)=A_1\ e^{i\delta_{\rm W_1}}e^{i\delta_{\rm S_1}}+A_2\ e^{i\delta_{\rm W_2}}e^{i\delta_{\rm S_2}}\ .$$

Then the corresponding amplitude for the decay $K^-\to f^-$ is

$$A(K^-\to f^-)=A_1\ e^{-i\delta_{\rm W_1}}e^{i\delta_{S_1}}+A_2\ e^{-i\delta_{\rm W_2}}e^{i\delta_{\rm S_2}}\ .$$

That is, the strong rescattering phases are the same but one complex conjugates the weak amplitudes. From the above, one sees that the rate asymmetry between these processes is

$$\begin{aligned}\mathcal{A}(f^+;f^-)&=\frac{\Gamma(K^+\to f^+)-\Gamma(K^+\to f^-)}{\Gamma(K^+\to f^+)+\Gamma(K^-\to f^-)}\\&=\frac{2A_1A_2\sin(\delta_{\rm W_2}-\delta_{\rm W_1})\sin(\delta_{\rm S_2}-\delta_{\rm S_1})}{A_1^2+A_2^2+2A_1A_2\cos(\delta_{\rm W_2}-\delta_{\rm W_1})\cos(\delta_{\rm S_2}-\delta_{\rm S_1})}\ .\end{aligned}$$

Unfortunately, detailed calculations in the standard CKM paradigm for rate asymmetries and asymmetries in Dalitz plot parameters for various charged Kaon decays give quite tiny predictions. This can be qualitatively understood as follows. The ratio $A_2\sin(\delta_{\rm W_2}-\delta_{\rm W_1})/A_1$ is typically that of a Penguin amplitude to a weak decay amplitude and so is of order ϵ'/ϵ. Furthermore, because of the small phase space for $K\to3\pi$ decays, or because one is dealing with electromagnetic rescattering in $K\to\pi\pi\gamma$, the rescattering contribution suppress these asymmetries even more. Table 2 gives typical predictions, contrasting them to the expected reach of the Frascati Φ factory with $\int\mathcal{L}\ dt=10\ {\rm fb}^{-1}$. For the $K\to3\pi$ decays, Belkov *et al.* [42] give numbers at least a factor of 10 above those given in Table 2. However, these numbers are predicated on having very large rescattering phases which do not appear to be realistic [43]. One is lead to conclude that, if the CKM paradigm is correct, it is unlikely that one will see a CP-violating signal in charged Kaon decays.

2.2.3 $K_{\rm L}\to\pi^o\ell^+\ell^-$; $K_{\rm L}\to\pi^o\nu\bar{\nu}$

Perhaps more promising are decays of the $K_{\rm L}$ to π^o plus lepton pairs. If the lepton pair is charged, then the process has a CP conserving piece in which the decay proceeds via a 2γ intermediate state. Although there was some initial controversy [46], the rate for the process $K_{\rm L}\to\pi^o\ell^+\ell^-$ arising from the CP-conserving 2γ transitiion is probably at, or below, the 10^{-12} level [47]:

$$B(K_{\rm L}\to\pi^o\ell^+\ell^-)_{\rm CP\ cons.}=(0.3-1.2)\times10^{-12}.$$

Thus this contribution is just a small correction to the dominant CP-violating amplitude arising from an effective spin 1 virtual state, $K_{\rm L} \to \pi^o J^*$. Since $\pi^o J^*$ is CP even, this part of the amplitude is CP-violating. It has two distinct pieces [48]: an indirect contribution from the CP even piece (ϵK_1) in the $K_{\rm L}$ state, and a direct $\Delta S = 1$ CP-violating piece coming from the K_2 part of K_L:

$$A(K_{\rm L} \to \pi^o J^*) = \epsilon A(K_1 \to \pi^o J^*) + A(K_2 \to \pi^o J^*) \ .$$

To isolate the interesting direct CP contribution in this process requires understanding first the size of the indirect contribution. The amplitude $A(K_1 \to \pi^o J^*)$ could be determined absolutely if one had a measurement of the process $K_{\rm S} \to \pi^o \ell^+ \ell^-$. Since this is not at hand, at the moment one has to rely on various guesstimates. These give the following range for the indirect CP-violating branching ratio [49]

$$B(K_{\rm L} \to \pi^o \ell^+ \ell^-)^{\rm indirect}_{\rm CP\ violating} = (1.6 - 6) \times 10^{-12} \ ,$$

where the smaller number is the estimate coming from chiral perturbation theory, while the other comes from relating $A(K_1 \to \pi^o J^*)$ to the analogous amplitude for charged K decays.

The calculation of the direct CP-violating contribution to the process $K_{\rm L} \to \pi^o \ell^+ \ell^-$, as a result of electroweak Penguin and box contributions and their gluonic corrections, is perhaps the one that is most reliably known. The branching ratio obtained by Buras, Lautenbacher, Misiak and Münz in their next to leading order calculation of this process [50] is

$$B(K_{\rm L} \to \pi^o \ell^+ \ell^-)^{\rm direct}_{\rm CP-violating} = (5 \pm 2) \times 10^{-12} \ ,$$

where the error arises mostly from the imperfect knowledge of the CKM matrix.

Experimentally one has the following 90% C.L. for the two $K_{\rm L} \to \pi^o \ell^+ \ell^-$ processes:

$$\begin{aligned} B(K_{\rm L} \to \pi^o \mu^+ \mu^-) &< 5.1 \times 10^{-9} \\ B(K_{\rm L} \to \pi^o e^+ e^-) &< 1.8 \times 10^{-9} \end{aligned}$$

The first limit comes from the E799 experiment at Fermilab [51], while the second limit combines the bounds obtained by the E845 experiment at Brookhaven [52] and the E799 Fermilab experiment [53]. Forthcoming experiments at KEK and Fermilab should be able to improve these limits by at least an order of magnitude[7], if they can control the dangerous background arising from the decay $K_{\rm L} \to \gamma\gamma e^+ e^-$ [54]. Even more distant experiments in the future may actually reach the level expected theoretically for the $K_{\rm L} \to \pi^o e^+ e^-$ rate [55]. However, it will be difficult to unravel the direct CP-violating contribution from the indirect CP-violating contribution, unless the $K_{\rm S} \to \pi^o e^+ e^-$ rate is also measured simultaneously.

In this respect, the process $K_{\rm L} \to \pi^o \nu\bar{\nu}$ is very much cleaner. This process is purely CP-violating, since it has no 2γ contribution. Furthermore, it has a tiny indirect CP contribution, since this is of order ϵ times the already small $K^+ \to \pi^+ \nu\bar{\nu}$ amplitude [56]. Next to leading QCD calculations for the direct rate have been carried out by Buchalla and Buras [57], who give the following approximate formula for the branching ratio for this process

$$B(K_{\rm L} \to \pi^o \nu\bar{\nu}) = 8.2 \times 10^{-11} A^4 \eta^2 \left(\frac{m_t}{M_{\rm W}}\right)^{2.3} \ .$$

This value is very far below the present 90% C.L. obtained by the E799 experiment at Fermilab [58]

$$B(K_{\rm L} \to \pi^o \nu\bar{\nu}) < 5.8 \times 10^{-5} \ .$$

KTeV should be able to lower this bound substantially, perhaps to the level of 10^{-8}, but this still leaves a long way to go!

[7] The goal of the KEK 162 experiment is to get to a BR of $O(10^{-10})$ for this mode, while KTeV hopes to push this BR down to 5×10^{-11}.

2.2.4 $K^+ \to \pi^+\nu\bar{\nu}$

The last process I would like to consider in this section is the charged Kaon analogue to the process just discussed. Although the decay $K^+ \to \pi^+\nu\bar{\nu}$ is not CP violating, it is sensitive to $|V_{\rm td}|^2 \simeq A^2\lambda^6[(1-\rho)^2+\eta^2]$ and so, indirectly, it is sensitive to the CP violating CKM parameter η. For the CP violating decay $K_{\rm L} \to \pi^o\nu\bar{\nu}$ the electroweak Penguin and box contributions are dominated by loops containing top quarks. Here, because one is not looking at the imaginary part, one cannot neglect altogether the contribution from charm quarks. If one could do so, the branching ratio formula for $K^+ \to \pi^+\nu\bar{\nu}$ would be given by an analogous formula to that for $K_{\rm L} \to \pi^o\nu\bar{\nu}$ but with $\eta^2 \to \eta^2 + (1-\rho)^2$.

Because m_t is large, the $K^+ \to \pi^+\nu\bar{\nu}$ branching ratio is not extremely sensitive to the contribution of the charm-quark loops [59]. Furthermore, when next to leading QCD corrections are computed the sensitivity of the result to the precise value of the charm-quark mass is reduced considerably [60]. Buras *et al.* [61] give the following approximate formula for the $K^+ \to \pi^+\nu\bar{\nu}$ branching ratio

$$B(K^+ \to \pi^+\nu\bar{\nu}) = 2\times 10^{-11} A^4 \left[\eta^2 + \frac{2}{3}(\rho^e-\rho)^2 + \frac{1}{3}(\rho^\tau-\rho)^2\right]\left(\frac{m_t}{M_{\rm W}}\right)^{2.3} .$$

In the above the parameters ρ^e and ρ^τ differ from unity because of the presence of the charm-quark contributions. Taking $m_t = 175$ GeV and $m_c(m_c) = 1.30 \pm 0.05$ GeV [62], Buras *et al.* [61] find that ρ^e and ρ^τ lie in the ranges

$$1.42 \leq \rho^e \leq 1.55\ ;\ \ 1.27 \leq \rho^\tau \leq 1.38\ .$$

Using the range of η and ρ determined by the CKM analysis discussed earlier gives about a 40% uncertainty for the $K^+ \to \pi^+\nu\bar{\nu}$ branching ratio, leading to the expectation

$$B(K^+ \to \pi^+\nu\bar{\nu}) = (1 \pm 0.4)\times 10^{-10}\ .$$

This number is to be compared to the best present limit coming from the E787 experiment at Brookhaven. Careful cuts must be made in the accepted π^+ range and momentum to avoid potentially dangerous backgrounds, like $K^+ \to \pi^+\pi^o$ and $K^+ \to \mu^+\pi^o\nu$. There is a new preliminary result for this branching ratio [63]

$$B(K^+ \to \pi^+\nu\bar{\nu}) < 3\times 10^{-9} \quad (90\%\ \text{C.L.})$$

which updates the previously published result from the E787 collaboration [64]. This value is still about a factor of 30 from the interesting CKM model range, but there are hopes that one can get close to this sensitivity in the present run of this experiment.

2.3 The Promise of B-decay CP Violation

If the CKM paradigm is correct, the analysis of current constraints on the CKM matrix shows that the CP-violating phase δ is sizable. The reason why one has **small** CP-violating effects in the Kaon sector is solely due to the interfamily mixing suppression. In B_d and B_s decays one can involve all three generations directly and, in certain cases, one can obviate this supression altogether [65]. Thus the study of CP violation in B-decays appears very promising.

To produce CP-violating effects, as usual, one has to have interference between two amplitudes that have different CP-violating phases. Because one is interested in looking for potentially large sources of CP violation in B-decays, it is important to identify where in the B system sizable phases may reside. Within the CKM paradigm there are two places where large phases appear. The first of these is the relative phase between the $|B_d>$ and $|\bar{B}_d>$ states, which make up the neutral B mass eigenstates:

$$|B_{d\pm}> \simeq \frac{1}{\sqrt{2}}[(1+\epsilon_{B_d})|B_d> \pm (1-\epsilon_{B_d})|\bar{B}_d>]\ .$$

This $B_d - \bar{B}_d$ mixing phase arises from a box-graph similar to that of Fig. 1a. Here, however, this graph is dominated by the contribution of the top quark loop and one has that

$$\frac{(1-\epsilon_{B_d})}{(1+\epsilon_{B_d})} \simeq \frac{V_{td}}{V_{td}^*} = e^{-2i\beta} ,$$

where β is the phase of the td matrix element of the CKM matrix ($V_{td} = |V_{td}|e^{-i\beta}$). If the CKM phase δ is large so is, in general, β since

$$\tan\ \beta = \frac{\sigma \sin\ \delta}{1-\sigma\cos\ \delta} = \frac{\eta}{1-\rho} .$$

The second place where a large phase appears is in any process involving, at the quark level, a $b \to u$ transition since $V_{ub} = |V_{ub}|e^{-i\delta}$ provides precisely a meausure of the CKM phase.

The potentially large $B_d - \bar{B}_d$ CP-violating phase β is a prediction of the CKM paradigm which can be well tested, since this phase is unpolluted by strong interaction effects. This is also the case for the corresponding mixing phase arising from $B_s - \bar{B}_s$ mixing. However, in this case the CKM prediction is that this phase should vanish, since V_{ts} is approximately real. So for the case of B_s decays the relevant tests are null tests. At any rate, because of $B_d - \bar{B}_d$ mixing, a state $|B_{d\ \rm phys}(t) >$ which at $t = 0$ was a pure $|B_d >$ state, evolves in time into a superposition of $|B_d >$ and $|\bar{B}_d >$ states:

$$|B_{d\ \rm phys}(t) >= e^{-im_{B_d}t}e^{-\Gamma_{B_d}t/2}[\cos\Delta m_d t/2\ |B_d > +ie^{-2i\beta}\sin\Delta m_d t/2\ |\bar{B}_d >] .$$

A similar formula applies for B_s decays.

It is also possible to isolate cleanly the CP-violating phases appearing at the quark level by comparing the decays of B mesons into some definite final state f to the corresponding transition of $\bar{B}$ mesons to the charged-conjugate final state $\bar{f}$. If these transitions are dominated by a single quark decay amplitude [66], so that

$$A(B \to f) = a_f e^{i\delta_f}; \qquad A(\bar{B} \to \bar{f}) = a_f^* e^{i\delta_f} ,$$

where δ_f is a strong rescattering phase, then the ratio of these two amplitudes will be directly sensitive to the quark decay phase. In these circumstances, for decays involving $b \to u$ transitions, the ratio

$$\frac{A(\bar{B} \to \bar{f})}{A(B \to f)} \simeq \frac{A(b \to uq\bar{q}')}{A(\bar{b} \to \bar{u}\bar{q}q')} = \frac{V_{ub}}{V_{ub}^*} = e^{-2i\delta}$$

is a measure of the CP-violating phase δ. On the other hand, the corresponding ratio of amplitudes which involve a $b \to c$ transition at the quark level will contain no large CP-violating phase at all, since V_{cb} is real.

In view of the above considerations, the best way to study CP violation in neutral B-decays is through a comparison of the time evolution of decays of states "born" as a B_d (or a B_s) into final states f, which are CP self-conjugate [$\bar{f} = \pm f$], to the corresponding time evolution of states which were born as a $\bar{B}_d$ (or a $\bar{B}_s$) and decay to $\bar{f}$ [67]. A straightforward calculation gives for the time dependent rates for these processes the expressions:

$$\Gamma(B_{phys}(t) \to f) = \Gamma(B \to f)e^{-\Gamma_B t}[1 - \eta_f\lambda_f \sin\Delta m_B t]$$

$$\Gamma(\bar{B}_{phys}(t) \to \bar{f}) = \Gamma(B \to f)e^{-\Gamma_B t}[1 + \eta_f\lambda_f \sin\Delta m_B t]$$

In the above η_f characterizes the CP parity of the state f, with $\bar{f} = \eta_f f$ and $\eta_f = \pm 1$, while λ_f encapsulates the mixing and decay CP violation information for the process. For all decays $B \to f$ which are dominated by just **one** weak decay amplitude the parameter λ_f is free of strong interaction complications and takes one of four values, depending on whether one is dealing with a B_d or B_s decay and on whether the decay processes at the quark level involves a $b \to c$ or a $b \to u$ transition. In these circumstances λ_f meausures purely CKM information and one finds

$$\begin{array}{ll} \lambda_f = \sin 2\beta & [B_d \text{ decays; b} \to \text{c transition}] \\ \lambda_f = \sin 2(\beta+\delta) \equiv \sin 2\alpha & [B_d \text{ decays; b} \to \text{u transition}] \\ \lambda_f = 0 & [B_s \text{ decays; b} \to \text{c transition}] \\ \lambda_f = \sin 2\delta \equiv \sin 2\gamma & [B_s \text{ decays; b} \to \text{u transition}] \ . \end{array}$$

The angles α, β and γ entering in the above equations have a very nice geometrical interpretation [68]. They are the angles of the, so called, unitarity triangle in the $\rho - \eta$ plane, whose base is along the ρ-axis going from $\rho = 0$ to $\rho = 1$ and whose apex is the point (ρ, η). That this is the case can be easily deduced by considering the bd matrix element of the CKM unitarity equation ($V^\dagger V = 1$):

$$V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0 \ .$$

To leading order in λ, the above equation reduces to

$$V_{ub}^* + V_{td} = A\lambda^3$$

which, upon dividing by $A\lambda^3$, is precisely the equation describing the unitarity triangle. One possible unitarity triangle, with the angles α, β and γ identified, is shown in Fig. 3.

Because our present knowledge of the CKM matrix still allows a considerable range for ρ and η, as shown in Fig. 3, there is considerable uncertainty on what to expect for $\sin 2\alpha$, $\sin 2\beta$ and $\sin 2\gamma$. Nevertheless, from an analysis of the allowed region in the $\rho - \eta$ plane, one can infer the allowed ranges for the unitarity triangle angles. In general, one finds that while both $\sin 2\alpha$ and $\sin 2\gamma$ can vanish, $\sin 2\beta$ is both **nonvanishing** and **large** [70]. This is illustrated in Fig. 4, taken from my recent examination of this question with Wang [25], which plots the presently allowed region in the $\sin 2\alpha - \sin 2\beta$ plane. One sees from this figure that

$$0.23 \leq \sin 2\beta \leq 0.84 \ .$$

Thus, in contrast to CP violation phenomena in the Kaon system, for the B system within the CKM paradigm there are places where one expects to see large effects.

From the above discussion, it is clear that a particularly clean test of the CKM paradigm would be provided by the meausurement of $\sin 2\beta$, via the observation of a difference in the rates of specific B_d and $\bar{B}_d$ decays to CP self-conjugate states which involve a $b \to c$ transition. A favored mode to study is the decay $B_d \to \psi K_S$ along with its conjugate [71]. This decay has a largish branching ratio [3] and quite a distinct signature from the leptonic decay of the ψ. Furthermore, one can argue that this decay is quite clean theoretically. Recall that the identification of λ_f with one of the angles of the unitarity triangle required that the decay rate for the process $B \to f$ be dominated by a single decay amplitude. This, in general, is only approximately true. For instance, for the process in question, at the quark level both a $b \to c$ decay graph **and** a $b \to s$ Penguin graph contribute. However, although this decay involves more than one amplitude, both of these amplitudes have the **same** weak decay phase [72]. The amplitude involving the quark decay graph has no weak phase since it involves a $b \to c$ transition. This is also true for the $b \to s$ Penguin graph, since this graph is dominated by the top loop contribution which is dominantly real. Hence, effectively, the ratio of $A(\bar{B}_d \to \psi K_S)$ to $A(B_d \to \psi K_S)$ is, to a very good approximation, unity and $\lambda_{\psi K_S}$ indeed meausures $\sin 2\beta$.

Even though $\sin 2\beta$, at least in the CKM paradigm, is large, the meausurement of $\lambda_{\psi K_S}$ is far from trivial, since one must be able to determine whether the decaying state was born as a B_d or a $\bar{B}_d$ and one must have sufficient rate to detect the produced ψ though its small leptonic decay mode. Nevertheless, it is clear that future meausurements of this and allied decay modes (like $B_d \to \psi K^* \to \psi K_S \pi$ [73]) perhaps at HERA-B, but certainly at the B factories under construction at SLAC and KEK and eventually in hadron colliders, offers an excellent chance of verifying- or put into question- the CKM paradigm.

The prospects of meausuring the other two angles of the unitarity triangle, α and γ, and thus of checking the CKM prediction $\alpha + \beta + \gamma = \pi$, appear more difficult. These meausurments

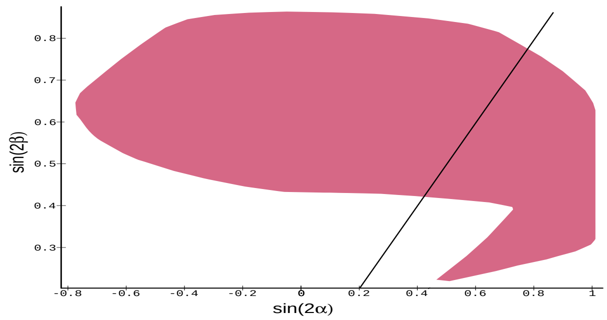

Figure 4: Plot of the allowed region in the $\sin 2\alpha - \sin 2\beta$ plane. The vertical line corresponds to the superweak prediction $\sin 2\alpha = \sin 2\beta$ [69)].

are, nevertheless, quite important. As Winstein [69)] has pointed out, even if one were to meausure a large value for $\sin 2\beta$ this still does not totally exclude a superweak explanation. One could imagine a perverse superweak model which, somehow, contained a small $\Delta S = 2$ CP-violating mixing phase, of $O(\epsilon)$, but a large $\Delta B = 2$ CP-violating mixing phase, 2β. This model can be ruled out by meausuring the angle α independently since, as there are no decay phases, it predicts that $\sin 2\beta = \sin 2\alpha$. As Fig. 4 shows, this "superweak' prediction is not excluded by present day data. Thus, if by chance, future meausurements of α and β were to fall on the superweak line one would still need a measurement of γ to settle the issue [8].

Most likely, the decay mode $B_d \to \pi^+\pi^-$ is the process that is best suited to study the angle α [71)]. However, the branching ratio for this mode is not yet totally in hand, but is probably quite small, of $O(10^{-5})$ [9]. This mode also may suffer some Penguin pollution, with estimates ranging from 1 % to 10 % in the amplitude [72)]. The CLEO [74)] indications that the branching ratio of $B_d \to \pi K$ is approximately the same as that for $B_d \to \pi\pi$ give one already some assurances that the Penguin amplitude in $B_d \to \pi^+\pi^-$ cannot dominate the process, since this amplitude is smaller by a factor of $|V_{td}|/|V_{ts}|$ compared to the $B_d \to \pi K$ amplitude. Furthermore, in principle, one can isolate the contribution of the Penguin graphs in the process $B_d \to \pi^+\pi^-$ by meausuring in addition the decays $B_d \to \pi^o\pi^o$ and $B^+ \to \pi^+\pi^o$ [75)]. These extra meausurements allow for a complete isospin analysis of the amplitudes entering in these two body decays and an isolation of the relative rescattering phases. Similar techniques [76)] may allow the extraction of the angle α in other decay modes, like $B_d \to \rho\pi$, which are not CP self-conjugate.

The determination of the angle γ is even more problematic. In principle, this angle can be determined by studying the time evolution of certain B_s decays. Here, perhaps, the best mode to study would be the decay $B_s \to \pi^o K_S$. However, at the asymmetric B factories now under construction, meausurements of B_s decays are unlikely. These colliders are optimized to operate at the $\Upsilon(4s)$ and running above the B_s production threshold will entail substantial loss of luminosity. Thus a determination of $\sin 2\gamma$ from B_s decays will have to await dedicated experiments at hadron colliders. One may, however, be able to determine γ, and thus the CKM phase δ, before that by utilizing different techniques. For instance, Gronau and Wyler [77)], have shown that one could, in principle, extract γ by studying the charged B decays $B^\pm \to DK^\pm$ and their neutral counterparts, using isospinology to isolate the rescattering phases. It remains to be seen, however, whether this

[8] In superweak models the angle 2γ is the phase associated with $B_s - \bar{B}_s$ mixing and does not necessarily vanish. However, in these models all B_s decays to CP self-conjugate states, irrespective of whether they involve a $b \to u$ or a $b \to c$ transition, should produce $\lambda_f = \gamma$.

[9] Recently, CLEO has observed a few B_d decays into pairs of light mesons and has been able to determine the branching ratio for the sum of the decay rates into both πK and $\pi\pi$ final states: $BR(B_d \to \pi K + \pi\pi) = (1.8 \pm 0.6 \pm 0.2) \times 10^{-5}$ [74)].

approach can really bear fruit in the presence of experimental errors.

3 Looking for new CP-violating phases

I would like to argue now a little more broadly about tests of CP violation. Obviously, it is very important to check whether the CKM paradigm is correct. Positive signals for $\epsilon'/\epsilon \neq 0$ will indicate the general validity of the CKM picture, since they require the presence of a $\Delta S = 1$ phase. However, given the large theoretical uncertainty on the value of this quantity, it is clear that values of ϵ'/ϵ consistent with zero at the 10^{-4} level cannot disprove this picture. In my view, it is likely that what will provide the crucial smoking gun for the CKM paradigm are searches for CP violation in the B system. The detection of the expected large asymmetry in $B_d \to \psi K_{\rm S}$ decays is of paramount importance, with the meausurement of the other angles in the unitarity triangle and of rare Kaon decays providing eventually a more detailed picture. However, whether the CKM picture is (essentially) correct or not, it is also importat to mount experiments which may provide the first glimpse at **other** CP-violating phases, besides the CKM phase δ. Of course, if the CKM picture is incorrect then one knows that at least some of these new phases are superweak in nature, arising in the $\Delta S = 2$ (and, perhaps, in the $\Delta B = 2$) sectors. Even if the CKM paradigm is essentially correct, there may be other phases which produce small violations in the fermion mixing matrix but which are important elsewhere.

Indeed, there are good theoretical arguments for having further CP-violating phases, besides the CKM phase δ. For instance, to establish a matter-antimatter asymmetry in the Universe one needs to have processes which involve CP violation [6]. If the origin of this asymmetry comes from processes at the GUT scale, then, in general, the GUT interactions contain further CP-violating phases besides the CKM phase δ [78]. If this asymmetry is established at the electroweak scale [79], then most likely one again needs further phases, both because intrafamily suppression gives not enough CP violation in the CKM case to generate the asymmetry and because one needs to have more than one Higgs doublet [80]. Indeed this last point gives the fundamental reason why one should expect to have further CP-violating phases, besides the CKM phase δ. It is likely that the standard model is part of a larger theory. For instance, supersymmetric extensions of the SM have been much in vogue lately. Any such extensions will introduce further particles and couplings and thus, by the simple corollary mentioned in the Introduction, they will introduce new CP-violating phases.

The best place to look for non-CKM phases is in processes where CP violation within the CKM paradigm is either vanishing or very suppressed. One such example is provided by experiments aimed at measuring the electric dipole moments of the neutron or the electron, since electric dipole moments are predicted to be extremely small in the CKM model. Another example concerns measurements of the transverse muon polarization $\langle P_\perp^\mu \rangle$ in $K_{\mu 3}$ decays, which vanishes in the CKM paradigm [81]. The transverse muon polarization measures a T-violating triple correlation [82]

$$\langle P_\perp^\mu \rangle \sim \langle \vec{s}_\mu \cdot (\vec{p}_\mu \times \vec{p}_\pi) \rangle \ .$$

In as much as one can produce such an effect also as a result of final state interactions (FSI) this is not a totally clean test for new CP-violating phases. With two charged particles in the final state, like for the decay $K_{\rm L} \to \pi^- \mu^+ \nu_\mu$, one expects FSI to give typically $\langle P_\perp^\mu \rangle_{\rm FSI} \sim \alpha/\pi \sim 10^{-3}$ [83]. However, for the process $K^+ \to \pi^o \mu^+ \nu_\mu$ with only one charged particle in the final state, the FSI effects should be much smaller. Indeed, Zhitnitski [84] estimates for this proceses that $\langle P_\perp^\mu \rangle_{\rm FSI} \sim 10^{-6}$. So a $\langle P_\perp^\mu \rangle$ measurement in the $K^+ \to \pi^o \mu^+ \nu_\mu$ decay is a good place to test for additional CP-violating phases.

The transverse muon polarization $\langle P_\perp^\mu \rangle$ is particularly sensitive to scalar interactions and thus is a good probe of CP-violating phases arising from the Higgs sector [85]. One can write the effective $K_{\mu 3}$ amplitude [86] as

$$A = G_{\rm F} \sin\theta_c f_+(q^2) \left\{ p_K^\mu \bar{\mu} \gamma_\mu (1-\gamma_5) \nu_\mu + f_S(q^2) m_\mu \bar{\mu} (1-\gamma_5) \nu_\mu \right\} \ .$$

Then

$$\langle P_\perp^\mu \rangle = \frac{m_\mu}{M_{\rm K}} {\rm Im}\ f_{\rm S} \left[\frac{|\vec{p}_\mu|}{E_\mu + |\vec{p}_\mu| n_\mu \cdot n_\nu - m_\mu^2/M_{\rm K}} \right] \simeq 0.2\ {\rm Im}\ f_{\rm S} \ .$$

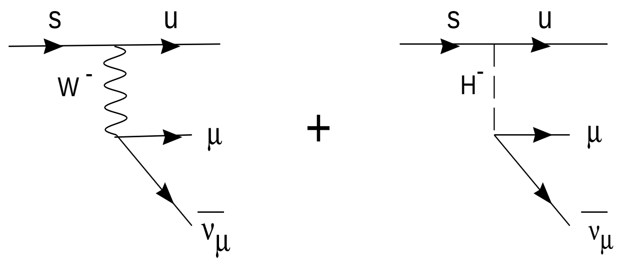

Figure 5: Graphs contributing to $\langle P_\perp^\mu \rangle$

Here $n_\nu (n_\nu)$ are unit vectors along the muon (neutrino) directions and the numerical value represents the expectation after doing an average over phase space [87].

Contributions to Im $f_{\rm S}$ can arise in multi-Higgs models (like the Weinberg 3-Higgs model [88]) from the charged Higgs exchange shown in Fig. 5, leading to [89]

$$\text{Im } f_{\rm S} \simeq \text{Im}(\alpha^* \gamma) \frac{M_{\rm K}^2}{M_{\rm H^-}^2} \; .$$

Here $\alpha(\gamma)$ are constants associated with the charged Higgs coupling to quarks (leptons). Because a leptonic vertex is involved, one in general does not have a strong constraint on $\text{Im}(\alpha^*\gamma)$. By examining possible non-standard contributions to the B semileptonic decay $B \to X \tau \nu_\tau$, Grossman [90] obtains

$$\text{Im}(\alpha^* \gamma) < \frac{0.23 \; M_{H^-}^2}{[\text{GeV}]^2}$$

which yields a bound for $\langle P_\perp^\mu \rangle$ of $\langle P_\perp^\mu \rangle < 10^{-2}$. Amusingly, this is the same bound one infers from the most accurate measurement of $\langle P_\perp^\mu \rangle$ done at Brookhaven about a decade ago [91], which yielded

$$\langle P_\perp^\mu \rangle = (-3.1 \pm 5.3) \times 10^{-3} \; .$$

In specific models, however, the leptonic phases associated with charged Higgs couplings are correlated with the hadronic phases. In this case, one can obtain more specific restrictions on $\langle P_\perp^\mu \rangle$ due to the strong bounds on the neutron electric dipole moment. For instance, for the Weinberg 3 Higgs model, one relates $\text{Im}(\alpha^*\gamma)$ to a similar product of couplings of the charged Higgs to quarks [89]:

$$\text{Im}(\alpha^* \gamma) = \left(\frac{v_u}{v_e} \right)^2 \; \text{Im}(\alpha^* \beta) \; ,$$

where v_u (v_e) are the VEV of the Higgs doublets which couples to up-like quarks (leptons). The strong bound on the neutron electric dipole moment [3] then gives the constraint

$$\text{Im}(\alpha^* \beta) \leq \frac{4 \times 10^{-3} \; M_{\rm H^-}^2}{[\text{GeV}]^2} \; .$$

If one assumes that $v_u \sim v_e$, this latter bound gives a strong constraint on $\langle P_\perp^\mu \rangle$ [$\langle P_\perp^\mu \rangle < 10^{-4}$]. However, this constraint is removed if $v_u/v_e \sim m_t/m_\tau$.

Similar results are obtained in the simplest supersymmetric extension of the SM. In this case, Im $f_{\rm S}$ arises from a complex phase associated with the gluino mass. Assuming all supersymmetric masses are of the same order, Christova and Fabbrichesi [92] arrive at the estimate

$$\text{Im } f_{\rm S} \simeq \frac{M_{\rm K}^2}{m_{\tilde g}^2} \frac{\alpha_{\rm s}}{12\pi} \sin \phi_{\rm susy} \; ,$$

where $\phi_{\rm susy}$ is the gluino mass CP-violating phase. This phase, however, is strongly restricted by the neutron electric dipole moment. Typically, one finds [93]

$$\sin \phi_{\rm susy} \leq \frac{10^{-7} \; m_{\tilde g}^2}{[\text{GeV}]^2}$$

leading to a negligible contribution for $\langle P_\perp^\mu \rangle$, below the level of $\langle P_\perp^\mu \rangle_{\rm FSI}$.

An experiment (E246) is presently underway at KEK aimed at improving the bound on $\langle P_\perp^\mu \rangle$ obtained earlier at Brookhaven. The sensitivity of E246 is such that one should be able to achieve an error $\delta \langle P_\perp^\mu \rangle \sim 5 \times 10^{-4}$ [87]. This level of precision is very interesting and, in some ways, it is comparable or better to d_n measurements for probing CP-violating phases from the scalar sector. This is the case, for instance, in the Weinberg model if v_u/v_e is large. At any rate, if a positive signal were to be found, it would be a clear indication for a non-CKM CP-violating phase. Furthermore, as Garisto [94] has pointed out, a positive signal at the level aimed by the E246 experiment would imply very large effects in the corresponding decays in the B system involving τ-leptons (processes like $B^+ \to D^o \tau^+ \nu_\tau$), since one expects, roughly,

$$\langle P_\perp^\tau \rangle_{\rm B} \sim \frac{M_{\rm B}}{M_{\rm K}} \frac{m_\tau}{m_\mu} \langle P_\perp^\mu \rangle_{\rm K} \ .$$

Thus, in principle, a very interesting experimental cross-check could be done.

4 Concluding Remarks

I would like to conclude more or less in the way in which I started this review, by reemphasizing that even after thirty years from the discovery of CP violation this phenomena remains shrouded in mystery. However, there are some grounds for optimism. It is quite possible that before the year 2000 we shall know whether the CKM model provides the approximately correct description of CP violation. For instance, a convincing non zero determination of ϵ'/ϵ would exclude the superweak hypothesis, while a meausurement of $\sin 2\beta \sim O(1)$ would strongly favor the CKM explanation. Both of these experimental results could be on hand in this time frame.

A more detailed understanding of the full CKM structure, or a further understanding of CP violation if the CKM paradigm fails, will be more difficult. The meausurement of α is probably the simplest of the more difficult things to accomplish. A direct meausurement of the CKM phase δ, or equivalently the angle γ, by means of experiments in the B sector or through the study of rare K decays, is very challenging. So are attempts at finding non CKM phases, although experiments searching for these effects are to be encouraged since they would signal new physics beyond the Standard Model. Indeed, it is important also that other CP violation experiments where one expects very small effects in the Standard Model be pushed to their limits, as surprises may arise. A case in point is provided by searches for CP violation in charged K decays, or in K_S decays, to be carried out here at DAPHNE, where little is really known. It is unclear, however, whether all this experimental activity will be able to throw any light on the strong CP problem. Nevertheless, if we are to really understand CP violation, one day we will have to understand why $\bar{\theta} \simeq 0$.

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